Mark Scheme (Results)

June 2011

GCE Statistics S4 (6686) Paper 1

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ¿ The answer is printed on the paper
   - □ The second mark is dependent on gaining the first mark

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June 2011
Statistics S4 6686
Mark Scheme

Question Number	Scheme	Marks
1.	$\mathrm{P}\left(F_{8,10} > 3.07\right) = 0.05$	
	So need $\mathrm{P}\left(F_{10,8} > x\right) = 0.01$ so $x = 5.81$	B1
	So $a = \frac{1}{5.81} =$ **0.172** awrt 0.172	B1
		2
2.	$s_p^{\ 2} = \frac{6s_x^{\ 2} + 3s_y^{\ 2}}{9}$ $(= 192.03\ldots)$	M1
	$1.735 < \frac{9s_p^{\ 2}}{\sigma^2} < 23.589$	B1M1B1
	So 99% confidence interval is $(73.26\ldots,\ 996.14\ldots)$ **awrt (73.3, 996)**	A1
		5
Notes:	1st M1 for attempting $s_p^{\ 2}$ 1st B1 for 1.735 (or better) 2nd M1 for use of $\frac{9s_p^{\ 2}}{\sigma^2}$, follow through their $s_p^{\ 2}$ 2nd B1 for 23.589 (or better) A1 for both values correct to awrt 3 sf	

Question Number	Scheme	Marks
3.	$d = B - A: 1, 2, 3, -1, 3, -1, -2, 2$	M1
	$\bar{d} = 0.875$	M1
	$s_d^{\ 2} = \dfrac{33 - 8\times 0.875^2}{7} = (3.8392\ldots)$	M1
	$H_0: \mu_d = 0 \qquad H_1: \mu_d > 0$	B1
	$t_7 = \dfrac{0.875}{\frac{s_p}{\sqrt{8}}} = 1.263\ldots$ **awrt <u>1.26</u>**	M1A1
	$t_7(10\%)$ one tail critical value is **<u>1.415</u>**	B1
	Not significant. There is insufficient evidence to support the claim of manufacturer *B* or machine *B* does not produce more juice (than machine *A*)	A1
		8
	1st M1 for attempting the *d*s 2nd M1 for attempting $\bar{d}$ 3rd M1 for attempting s_d or $s_d^{\ 2}$ 4th M1 for attempting the correct test statistic 3rd A1 contextual statement only required. Allow The juice provided by machine *A* is the same as by machine *B*	
	NB 2 sample test can score 3/8 M0 M0 M1 $\dfrac{7\times 9.27 + 7\times 16.79}{14}$ B1 for $H_0: \mu_A = \mu_B \quad H_1: \mu_A < \mu_B$ M0 A0 B1 1.345 A0	

Question Number	Scheme	Marks
4. **(a)**	[X = no. of incorrectly addressed letters. $X \sim B(40,0.05)$] $P(X > 3) = 1 - P(X \le 3), = 1 - 0.8619 = 0.1381$ **awrt <u>0.138</u>**	M1, A1 (2)
(b)	$P(\text{Type II Error}) = P(X \le 3 \mid p = 0.10)$ $= 0.4231$ **awrt <u>0.423</u>**	M1 A1 (2)
(c)	Power = 1 - P(Type II error) so $s =$ **<u>0.58</u>** (0.5769)	B1 (1)
(d)	Y = no. of incorrectly addressed letters in a sample of 15. $Y \sim B(15, 0.05)$ Size $= P(Y \ge 2) + P(Y = 1) \times P(Y \ge 2)$ $= [1 - 0.8290] \times [1 + 0.8290 - 0.4633]$ $= 0.23353\ldots$ **awrt <u>0.23</u>**	M1 A1 A1 (3)
(e)	(use overlay)	B1B1 (2)
(f)	2nd / consultants test is quicker (since it uses fewer letters) 2nd / consult test is more powerful for $p < 0.125$ (and values greater than this should be unlikely)	B1 B1 (2) **12**
Notes: **(a)** **(b)** **(c)** **(d)** **(e)** **(f)**	M1 for 1- $P(X \le 3)$ and $X \sim B(40, 0.05)$ M1 for a correct interpretation of P(Type II error) B1 must be 2dp M1 for a correct strategy 1st A1 for a correct numerical expression 1st B1 for correct points (accept $\pm$ one 2mm square) 2nd B1 for curve 1st B1 for selecting 2nd test 2nd B1 for a suitable supporting reason eg more powerful for small values of p/p around 0.05	

Question Number	Scheme	Marks
5.		
(a)	$s_x^2 = \dfrac{1559691 - 6\times\left(\dfrac{3059}{6}\right)^2}{5} = 22.1666...$	M1
	$H_0: \sigma_x^2 = \sigma_y^2 \quad H: \sigma_x^2 \neq \sigma_y^2$	B1
	$\dfrac{s_x^2}{s_y^2} = 1.895....$	M1
	$F_{5,4} = 6.26$	B1
	$\dfrac{s_x^2}{s_y^2} = 1.895....$ **awrt <u>1.90</u>** and comment : not significant - variances of **<u>weights</u>** of the two **<u>boxes</u>** can be assumed equal.	A1 (5)
(b)	$\bar{x} = 509.833... \Rightarrow \bar{x} - \bar{y} = 5.03333$	M1
	$s_p^2 = \dfrac{5s_x^2 + 4s_y^2}{9} = 17.513....$ **awrt <u>17.5</u>**	M1A1
	5% two tail t value is $t_9 = 1.833$	B1
	90% confidence interval is $5.03... \pm 1.833 \times \sqrt{17.513...} \times \sqrt{\frac{1}{6} + \frac{1}{5}}$	M1
	(0.388…., 9.6782…) **awrt <u>(0.388, 9.68)</u>**	A1, A1 (7)
(c)	Zero is not in CI, there <u>is</u> evidence to <u>reject</u> the manufacturer's claim Or the weight of the contents of the boxes has changed.	B1ft, B1ft (2) **14**
Notes: (a)	1st M1 for use of the correct formula for s_x^2 with reasonable attempt at $\sum x^2$ and $\sum x$ 2nd M1 for use of the correct test statistic. Allow use of 3.42 instead of 3.42^2. Top must be their variance.	
(b)	1st M1 for attempting $\bar{x} - \bar{y}$ can follow through their $\bar{x}$ 2nd M1 for attempt to find pooled estimate of variance 3rd M1 for use of correct formula for CI allow any t value and ft their $\bar{x}$ and s_p	

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Question Number	Scheme	Marks
6. **(a)**	$\mathrm{E}\left(Y^m\right)=\frac{n}{\beta^n}\int y^m \times y^{n-1}\,\mathrm{d}y$ =, $\left[\frac{n}{\beta^n}\times\frac{1}{m+n}\times y^{m+n}\right]_0^{\beta}$	M1, A1
	$=\frac{n}{\beta^n}\times\frac{1}{m+n}\times\beta^{m+n}=\frac{n}{m+n}\beta^m$ (*)	A1cso
		(3)
(b)	$\mathrm{E}(Y)=\frac{n}{n+1}\beta$	B1
		(1)
(c)	$\mathrm{E}\left(Y^2\right)=\frac{n}{n+2}\beta^2$, $\mathrm{Var}(Y)=\mathrm{E}\left(Y^2\right)-[\mathrm{E}(Y)]^2$	B1,M1
	$\mathrm{Var}(Y)=\frac{n}{n+2}\beta^2-\frac{n^2}{(n+1)^2}\beta^2=\frac{n}{(n+1)^2(n+2)}\beta^2$ (*)	A1cso
		(3)
(d)	As $n\to\infty$ $\mathrm{E}(Y)\to\beta$, $\mathrm{Var}(Y)\to 0$	M1,A1
	So Y is a consistent estimator for β.	A1
		(3)
(e)	$k=\frac{n+1}{n}$	B1
		(1)
(f)	$\mathrm{Var}(M)=4\mathrm{Var}(\bar{X})=4\frac{\sigma^2}{n}=\frac{4}{n}\times\frac{\beta^2}{12}=\frac{\beta^2}{3n}$	B1
	$\frac{(n+1)^2}{n^2}\times\frac{n}{(n+1)^2(n+2)}\beta^2=\frac{\beta^2}{n(n+2)}<\frac{\beta^2}{3n}$ so S is better $(n>1)$	M1A1
		(3)
(g)	Max = 9.1, $s=\frac{6}{5}\times 9.1$ = **<u>10.9(2)</u>**	M1A1
		(2)
		16

Question Number	Scheme	Marks
Notes:		
(a)	M1 for attempt to integrate $y^m \mathrm{f}(m)$ 1st A1 for correct integration (limits not needed yet) 2nd A1 for use of correct limits and proceeding to printed answer. No incorrect working seen.	
(c)	M1 for use of their E(Y) and E(Y^2) in a correct formula for Var(Y)	
(d)	M1 for examining both E(Y) and Var(Y) for $n \to \infty$ 1st A1 for correct limits for both the above 2nd A1 for a correct statement following correct working	
(f)	M1 for attempting Var(S)	
(g)	M1 for correct use of S to find estimate	
7. **(a)**	$s_x^2 = \dfrac{214856 - 20 \times \left(\dfrac{2072}{20}\right)^2}{19} = 10.357\ldots$ **awrt** <u>**10.4**</u>	B1
	$H_0: \sigma = 2.8$ (or $\sigma^2 = \ldots$) $H_1: \sigma \neq 2.8$ (or $\sigma^2 \neq \ldots$)	B1
	$\dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_{19}$ test statistic = 25.102… **awrt** <u>**25.1**</u>	M1A1
	$\chi^2_{19}(0.025) = 32.852$, $\chi^2_{19}(0.975) = 8.907$	B1B1
	Not significant so no evidence of a change in standard deviation	A1
		(7)

Question Number	Scheme	Marks
(b) (i)	$H_0 : \mu = 102.3 \quad H_1 : \mu \neq 102.3$	B1
	$z = \dfrac{\frac{2072}{20} - 102.3}{\frac{2.8}{\sqrt{20}}} = 2.0763...$ awrt **2.08**	M1A1
	Critical value is $z = 1.96$ or awrt $0.019 < 0.025$	B1
	So a significant result, there is evidence of a change in mean length	A1ft
(ii)	$t = \dfrac{\frac{2072}{20} - 102.3}{\sqrt{\frac{10.357...}{20}}} = 1.8064...$ awrt **1.81**	M1A1
	Critical value of $t_{19} = 2.093$	B1
	Not significant, there is insufficient evidence of a change in mean length	A1 (9)
(c)	(a) suggests that σ is unchanged so can use $\sigma = 2.8$ so normal test can be used	B1ft
	So using (i) conclude that there is evidence of an increase in mean length	B1ft (2) **18**
Notes: (a)	M1 for use of the correct test statistic	
(b)	1st and 2nd M1 for use of correct test statistics	
(c)	1st B1 for reason for selecting (i) or (ii) based on their conclusion from test in (a).	
	2nd B1 For a final conclusion about mean lengths based on their (a) and (b)	
	NB if both conclusions are the same it needs to be clear they have chosen (i)	

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